Nisho-98/3

Josephson Effects in Double-Layer Quantum Hall States

Aiichi Iwazaki

Department of Physics, Nishogakusha University, Shonan Ohi Chiba 277, Japan.

(April 15, 1998)

Abstract

Under quite plausible assumptions on double-layer quantum Hall states with strong interlayer correlation, we show in general framwork that coherent tunneling of a single electron between two layers is possible. It yields Josephson effects with unit charge tunneling. The origin is that Halperin states in the quantum Hall states are highly degenerate in electron number difference between two layers in the absence of electrons tunneling.

It has been shown [1,2] that in double-layer (DL) quantum Hall states with filling factor, $\nu = 1$, a zero energy mode arises owing to spontaneous symmetry breakdown of pseudo-spin U(1) symmetry. This symmetry is associated with the conservation of electron number difference between two layers. Thus the mode arises only for the system with no interlayer-tunneling; the tunneling leads obviously to non-conservation of the electron number difference and so breaks the symmetry explicitly. In Chern-Simons gauge theory this symmetry breakdown is realized [1,3] as a condensation of bosonized electrons; this is quite similar to the condensation of Cooper pairs in superconductors. Thus one working with the theory is led naturally to anticipate the existence of Josephson-like effects in the DL quantum Hall states. Actually it has been argued [1,3–5] by addressing the mode that Josephson effects may arise in these DL quantum Hall states. It has, however, been pointed

out [6] that the interlayer-tunneling gives a gap to the mode and freezes a phase degree of freedom whose existence is essential for Josephson effects. But this gap is owing to so-called Anderson plasmon [5] and is a universal property of standard Josephson junctions. Thus the phase coherence between two layers is still alive and Josephson effects are expected in the system.

In this paper using quite plausible assumptions we show that Josephson effects are possible in the DL quantum Hall states with $\nu = 1$. The assumptions are that in the case of no interlayer-tunneling, quantum Hall states in the DL system are described with Halperin states [7] and that such Halperin states are degenerate in the electron number difference between two layers, when small capacitance energies are neglected. Furthermore, we assume that an interaction describing interlayer tunneling conserves angular momentum (J) of electron; an electron with $J = m$ in a layer tunnels into a state with $J = m$ in the other layer. It is also assumed that the energy scale of the tunneling is much smaller than the Coulomb energy, e^2/l_B where l_B is the magnetic length and $-e$ is the charge of electron.

In the assumptions the degeneracy of Halperin states in the electron number difference is the consequence of the broken U(1) symmetry in the system with no interlayer-tunneling. Hence it seems to be quite acceptable. The assumption of angular momentum conservation is hold in the system with no irregularity which breaks the rotational symmetry. The last assumption on the energy scale guarantees that the tunneling effect can be treated perturbatively. Later we argue that possible deviations from these assumptions in realistic samples do not affect seriously the Josephson effects.

As has been shown numerically [8] in the $\nu = 1$ DL system with an appropriate interlayer distance, d, comparable to $l_B = \sqrt{1/eB}$ (B is magnetic field perpendicular to the layers), Halperin states are fairly good approximate ground states in the case of no interlayer-tunneling. Even in the presence of the interlayer-tunneling, the ground state may be approximated by mixture of the states.

First, we briefly sketch Halperin states with the filling factor $\nu = 1/m$ ($m =$ integer). They are described as

$$\Psi(N_1,N_2)=\prod_{l<k}(z_l-z_k)^m\prod_{s<t}(w_s-w_t)^m\prod_{l,s}(z_l-w_s)^m\exp(-\sum(|z|^2+|w|^2)/4l_B^2) \qquad (1)$$

where N_1 and N_2 are the numbers of electrons in each layer. We denote complex coordinates of electrons in each layer with z and w, respectively. These states are degenerate with each other in the electron number difference, N_1-N_2, when we neglect charging energies. Namely, when the filling factor $\nu=1/m$ is given, total number of electrons, N_1+N_2 is determined uniquely but the values of N_1-N_2 are arbitrary. This means that there are many quantum Hall states with $\nu=1$, which are characterized by quantum numbers, N_1-N_2. They are all degenerate. Once we take account of the charging energies,

$$H_c=\frac{e^2\hat{S}_z^{\ 2}}{C} \qquad (2)$$

the degeneracy is lifted up, where $C=\epsilon L^2/4\pi d$ is the electric capacitance of the double-layers; L^2 is the surface area of the layers and ϵ is the dielectric constant between two layers. The operator $\hat{S}_z$ represents electron number difference and its eigen value is $(N_1-N_2)/2$. Electric neutrality is assumed for the state with $N_1-N_2=0$. Obviously the charging energy is small enough for infinitely large L^2. Hence the Halperin states are almost degenerate even if the charging energy is switched on.

In the realistic DL samples, the states with $\nu=1$ have been realized [9,10]. Hence we only consider such states. First we note that in each layer, one particle states in the Lowest Landau level are characterized by the angular momentum, $J=m$ (m takes a value of 1 through $(N_1+N_2)/2$). Then, Halperin states with $\nu=1$ are states such that either of electrons in the first or second layer occupies each one particle eigenstate with $J=m$, but electrons in both layers do not occupy simultaneously the states with the same angular momentum. More explicitly, the state can be written such that

$$|Halperin>=P(N_1,N_2)\prod_{m=0}^{N_1+N_2-1}(a_m^\dagger+b_m^\dagger)|0> \qquad (3)$$

with $a_m|0>=b_m|0>=0$, where $P(N_1,N_2)$ is the projection operator picking up a state with N_1 electrons on the 1st layer and N_2 electrons on the 2nd layer. a_m and b_m are

annihilation operators of electrons with angular momentum, m, on the 1st layer and 2nd layer respectively.

When the interlayer-tunneling is allowed, these Halperin states are mixed with each other. We assume the following interlayer-tunneling interaction,

$$H_t = -\frac{\Delta_{sas}}{2}\int(\Psi_1^\dagger\Psi_2 + h.c.) = -\frac{\Delta_{sas}}{2}(\sum_m a_m^\dagger b_m + h.c. + higher\ Landau\ levels) \tag{4}$$

where Ψ_i is the electron field of i-th layer and Δ_{sas} is the energy difference of symmetric ($\Psi_1 + \Psi_2$) and antisymmetric ($\Psi_1 - \Psi_2$) states. This tunneling term preserves the angular momentum of electrons.

It is interesting to see that a Halperin state, $|S, S_z>$, which is an eigenstate of $\hat{S}_z$ with the eigenvalue of $(N_1 - N_2)/2$ ($S = (N_1 + N_2)/2$), is transformed to the state, $|S, S_z \pm 1>$ by the tunneling interaction,

$$H_t|S, S_z> = -\frac{\Delta_{sas}}{2}(\sqrt{(S - S_z)(S + S_z + 1)}|S, S_z> + \sqrt{(S + S_z)(S - S_z + 1)}|S, S_z - 1>). \tag{5}$$

It can be also derived by using $O(3)$ algebra, $[\hat{S}_i, \hat{S}_j] = i\epsilon_{ijk}\hat{S}_k$, where

$$\hat{S}_x = \frac{1}{2}\int(\Psi_1^\dagger\Psi_2 + h.c.),\ \hat{S}_y = \frac{-i}{2}\int(\Psi_1^\dagger\Psi_2 - \Psi_2^\dagger\Psi_1),\ and\ \hat{S}_z = \frac{1}{2}\int(\Psi_1^\dagger\Psi_1 - \Psi_2^\dagger\Psi_2). \tag{6}$$

In this notation $H_t = -\Delta_{sas}\hat{S}_x$. These operators, $\hat{S}_i$, are called pseudospin operators and Halperin states, $|S, S_z>$ are elements of a representation space of the operators. Namely they represent the states with the pseudospin, S, whose z component is S_z.

As far as we neglect the effects of the charging energy and the tunneling interaction, Halperin states $|S, S_z>$ are realized as quantum Hall states with $\nu = 1$. These states are degenerate in the quantum number, S_z. But once we include these effects, the degeneracy is lifted up. To find the groundstate, we need to diagonalize the Hamiltonian,

$$H = H_c + H_t = \frac{e^2\hat{S}_z^{\ 2}}{2C} - \Delta_{sas}\hat{S}_x \tag{7}$$

in the space of Halperin states, $|S, S_z>$. Thus we obtain an eigenstate of the Hamiltonian,

$$|G>=\sum_n A_n|S,n> \tag{8}$$

where A_n satisfies the recursion formula,

$$EA_n = \frac{e^2n^2}{2C}A_n - \frac{\Delta_{sas}}{2}[A_{n-1}\sqrt{(S-n+1)(S+n)} + A_{n+1}\sqrt{(S+n+1)(S-n)}], \tag{9}$$

with E being the energy of the eigenstate $|G>$. In order to solve the recursion formula, we assume $S=(N_1+N_2)/2$ being much larger than any n, in other words, A_n with n being the order of S is small enough to be neglected. Then we expand the square root, leaving the terms of the lowest order in n/S. Setting $\Psi(\theta)=\sum A_n e^{i\theta n}$, we rewrite the formula such that

$$E\Psi(\theta) = -\frac{e^2}{2C}\frac{\partial^2}{\partial\theta^2}\Psi(\theta) - S\Delta_{sas}\cos\theta\Psi(\theta). \tag{10}$$

This $\Psi(\theta)$ represents the wave function of the eigenstate $|G>$ in terms of angle variable θ conjugate to the electron number difference $n\sim N_1-N_2$ between two layers,

$$\Psi(\theta)=<\theta|G>, \tag{11}$$

with $|\theta>=\sum_n e^{-i\theta n}|S,n>$

A solution of this Shrödinger-like equation can be obtained by assuming a particle sitting in the bottom of the cosine potential, i.e. $\theta\ll 1$,

$$\Psi(\theta)\sim\exp(-mE_0\theta^2/2) \tag{12}$$

where $m=C/e^2$ is the mass of the particle and $E_0=\sqrt{e^2S\Delta_{sas}/C}$ is the energy of the state.

Here we comment that since θ is the conjugate variable to $S_z\sim N_1-N_2$, roughly it represents a direction of the pseudospin in x-y plane. Thus the above solution represents a state with the direction of the pseudospin pointed to x axis, i.e. $\theta=0$. This fact can be understood by noting the existence of the term, $\Delta_{sas}\hat{S}_x$ in the Hamiltonian; the term implies the imposition of magnetic field, $B=\Delta_{sas}$ pointed to x axis. Thus the spin is pointed to x axis.

We also comment that since the fluctuation of θ is given by $(mE_0)^{-1/2} \propto \Delta_{sas}^{1/2}$, it diverges in the limit of a vanishing tunneling amplitude $\sim \Delta_{sas}$ of a electron. Thus the phase θ is not well defined and Josephson phenomena are not expected to be seen in the limit just as in superconducting Josephson junctions.

As can be seen easily, there exist solutions representing the pseudospin rotating around z axis; the particle is not bounded to the cosine potential and it moves from $\theta = -\infty$ to $\theta = +\infty$. Obviously, this mode does not correspond to the groundstate of the system eq(7). But the mode is excited by applying a voltage, V_0 between two layers. To see it, we add a term, $eV_0\hat{S}_z$ to the Hamiltonian,

$$H' = H + eV_0\hat{S}_z = \frac{e^2\hat{S}_z{}^2}{2C} - \Delta_{sas}\hat{S}_x + eV_0\hat{S}_z \tag{13}$$

Then, it follows with the similar manipulation to the above one that the wave function $\Psi(\theta)$ satisfies the following equation,

$$(i\partial_t + ieV_0\partial_\theta)\Psi(\theta) = -\frac{e^2}{2C}\partial_\theta^2\Psi(\theta) - S\Delta_{sas}\cos\Psi(\theta), \tag{14}$$

where we have explicitly indicated the derivative in time, t, in order to see the evolution of Ψ. Assuming a wave packet, $\Psi(\theta)$, in θ, we can show that

$$\frac{d}{dt} < \theta >= \frac{d}{dt}\int \overline{\Psi}\theta\Psi d\theta = eV_0 + \frac{e}{C} < en >= eV_0 + eV_{ind} \equiv eV_{me}, \tag{15}$$

where $V_{ind} = e < n >$ denotes an induced voltage associated with the charging, $< en >$ and V_{me} does the voltage measured actually.

This implies that the phase, θ, conjugate to the electron number difference, $N_1 - N_2$, evolves according to the standard Josephson equation. In terms of the pseudospin languages, the spin rotates around z axis. Furthermore, we can show that the tunneling current is given by

$$\frac{d}{dt} < -en >= \frac{d}{dt}\int \overline{\Psi}ei\frac{\partial}{\partial\theta}\Psi d\theta = eS\Delta_{sas} < \cos\theta >\approx J_c\cos(<\theta>), \tag{16}$$

with $J_c = eS\Delta_{sas}$, where J_c is the critical current. Hence we can see that the current is the same as the one in standard Josephson effects; conventionally the current is given by $J_c\sin\theta$, which is obtained by shifting $\theta \to \theta + \pi/2$ in eq(16).

Consequently, we obtain the Josephson equations controlling the phase and the tunneling current in the quantum Hall system.

It is instructive to see the quantum Hall state Josephson effects in a different way. We may rewrite Hamiltonian eq(13) or equivalently Hamiltonian read from Schrödinger equation eq(14) as follows,

$$H' = \frac{p^2}{2m} + eV_0 p - S\Delta_{sas} \cos\theta, \tag{17}$$

with $p = -i\frac{\partial}{\partial\theta}$.

Using this Hamiltonian we find that the velocity of θ is given by $\dot{\theta} = p/m + eV_0$ where a dot denotes a time derivative. On the other hand a time variation of p is given by $\dot{p} = -S\Delta_{sas}\sin\theta$. Thus it follows that $m\ddot{\theta} = -S\Delta_{sas}\sin\theta$. This is the equation of pendulum. The only effect of switching on the voltage, V_0, changes the velocity of the pendulum by eV_0. Before applying the voltage its momentum p takes a value of the order of $\sqrt{mE_0}$ in the bottom of the cosine potential as shown in eq(12). Hence the velocity, $\dot{\theta} \sim \sqrt{E_0/m} \sim \sqrt{1/L^2}$, is quite small. However, once the voltage is switched on, the pendulum gains a velocity, eV_0, and so it can climb the mountain of the potential, when eV_0 is sufficiently large. Thus the pendulum can rotate as θ increases without limit; the pseudospin can rotate around z axis. In other words, Josephson effects arise owing to the voltage between two layers.

The essence of this phenomena, coherent interlayer tunneling of a single electron, is that Halperin states, $|S, n>$, are highly degenerate in their quantum number $n = (N_1 - N_2)/2$. Namely even if electrons move from a layer to the other one, the energy of the system does not change. The situation is quite similar to that of Josephson junctions, in which states composed of two superconducting states are degenerate in the difference of numbers of Cooper pairs involved in each superconducting state. That is, the energy of the system does not change even if Cooper pairs move from a superconductor to the other one. In both cases this degeneracy leads to the existence of the phase conjugate to the quantum number n; the phase becomes a good quantum number characterizing the quantum Hall state as well as the state of Josephson junction.

Only difference between the case of the quantum Hall states and that of Josephson junction is that in the DL quantum Hall states, there does not exist a phase degree of freedom associated with each state of the two layers, on the other hand there exist a phase degree of freedom associated with each state of two superconductors in Josephson junction.

As we have shown under the plausible and general assumptions, Josephson effects arise in the DL quantum Hall states with $\nu = 1$. Some of these assumptions have been confirmed numerically. In the realistic samples however these purely assumptions do not necessarily hold. But small deviations from the above hypothetic Hamiltonian or deviations from the Halperin states may be regarded as effects of impurities. These effects may not change our results because coherent phenomena like Josephson effects are not seriously affected by the impurities in general. Therefore we expect that the effects arise in the DL quantum Hall states.

Recent experiment [10] on tunneling currents between two layers (whose separations are much larger than ones claimed for the existence of Josephson effects) shows the existence of an exciton made of a tunneled electron and a hole it leaves behind; the energy of the exciton yields actual tunneling barrier. It suggests a state of the exciton with zero energy when the interlayer separation is sufficiently small, but nonvanishing. Then the tunneling barrier is expected to vanish for the system and electrons tunnel freely without friction. This may reads to Josephson effects in the DL quantum Hall states, as we have discussed in this paper.

The author would like to express thanks to members of particle theory group for their hospitality in tanashi, KEK.

REFERENCES

[1] Z.F. Ezawa and A. Iwazaki, Int. Mod. Phys. B6, 3205 (1992); Phys. Rev. B48, 15189 (1993), The zero mode has already been found by M. Rasolt, F. Perrot and A.H. MacDonald, Phys. Rev. Lett. 55 433 (1985).

[2] X.G. Wen and A. Zee, Phys. Rev. Lett. 69, 1811 (1992).

[3] Z.F. Ezawa and A. Iwazaki, Phys. Rev. B47, 7295 (1993), Z.F. Ezawa, A. Iwazaki and Y.S. Wu, Mod. Phys. Lett. 1223 (1993).

[4] X.G. Wen and A. Zee, Phys. Rev. B47, 2265 (1992).

[5] Z.F. Ezawa and A. Iwazaki, Int. J. Mod. Phys. B8, 2111 (1994).

[6] A.H. MacDonald and S.C. Zhang, Phys. Rev. B49, 17208 (1994).

[7] B.I. Halperin, Helv. Phys. Acta 56, 75 (1983).

[8] T. Nakajima and H. Aoki, Physica B201, 327 (1994).

[9] S.Q. Murphy etal. Phys. Rev. Lett. 72, 728 (1994), T.S. Lay etal. Phys. Rev. B50, 17725 (1994), A. Sawada etal. Solid State Commun. 103, 447 (1997)

[10] J.P. Eisenstein etal. Phys. Rev. Lett. 74, 1419 (1995).